Journal of Engineering Physics and Thermophysics, Vol. 74, No. 1, 2001

EFFECT OF THE COMPRESSIBILITY OF THE FLUID ON THE PARAMETERS OF A HYDRAULIC GUN

A. N. Semko

UDC 532.5:518.5

The effect of the compressibility of the fluid on the parameters of a hydraulic gun is evaluated. The quasi-one-dimensional motion of an ideal compressible fluid is described by equations of nonstationary gas dynamics, which are solved numerically according to the algorithm proposed. The numerical solution for a compressible fluid is compared with an analytical solution for an incompressible fluid and with an experiment, which are performed by other authors. From an analysis of the results conclusions are drawn that the compressibility of the fluid can be disregarded. It is proposed that the Mach number be used as a criterion for assessing account for the compressibility of the fluid.

For the production of high-speed pulsed jets of a fluid, a pulsed water jet (monitor) and a hydraulic gun are widely used. The first designs of a pulsed water jet and a hydraulic gun were developed and manufactured at the Institute of Hydrodynamics of the Siberian Branch of the Russian Academy of Sciences [1]. It is there that experimental and theoretical investigations of these installations were started, too [2]. The theory of the installations was developed within the framework of the model of an ideal incompressible fluid. Further experimental and theoretical investigations of hydropulsed installations [3] showed that disregarding the compressibility of the fluid may lead to significant errors. Certain special features of the steady motion of a compressible fluid are described in [4]. In [5], the flow of a free water charge of incompressible fluid into a narrowing nozzle of a hydraulic gun of arbitrary profile is considered analytically. In [6], a similar problem is solved numerically for a compressible fluid and a comparison with an incompressible fluid is made [5]. Experimental investigations of a hydraulic gun and a comparison with the calculations of [5, 6] are performed in [7]. In the present work, the conditions for disregarding fluid compressibility are evaluated in detail using different well-tested numerical methods.

The main parts of a hydraulic gun are a cylindrical barrel 1 and a narrowing nozzle 3 (Fig. 1). Let the water charge 2, moving in the barrel with the velocity U_0, reach the inlet to the nozzle and begin to flow into it at the initial instant of time. We will consider the fluid to be ideal and compressible and the profile of the nozzle to be assigned and smoothly changing; the radial flow of the fluid is disregarded. We will denote the inlet and outlet cross sections of the nozzle by F_{in} and F_{e} and its length by L_{e} and bring the origin of coordinates into coincidence with the rear edge of the water charge at the initial instant of time. In the adopted formulation, the quasi-one-dimensional motion of the ideal compressible fluid in the hydraulic gun is described by the system of equations of nonstationary gas dynamics

$$\frac{\partial \rho F}{\partial t} + \frac{\partial \rho u F}{\partial x} = 0 \,,$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \frac{n}{n-1}\frac{p+B}{\rho}\right) = 0 \,,$$

Donetsk State University, Donetsk, Ukraine; email: stupin@dongu.donetsk.ua. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 1, pp. 27–31, January–February, 2001. Original article submitted November 16, 1999; revision submitted August 1, 2000.

1062-0125/01/7401-0035$25.00 ©2001 Plenum Publishing Corporation

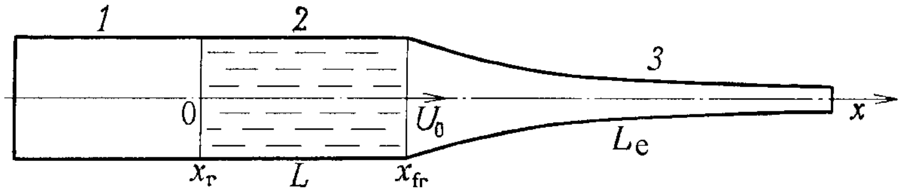

Fig. 1. Scheme of the hydraulic gun: 1) barrel; 2) water charge; 3) nozzle.

$$p = B\,[(\rho/\rho_0)^n - 1]\,. \tag{1}$$

The initial and boundary conditions for system (1) are as follows:

$$u\,(0, x) = U_0\,, \quad p\,(0, x) = 0\,, \quad \rho\,(0, x) = \rho_0\,; \quad 0 \le x \le L\,, \tag{2}$$

$$p\,(t, x_r) = 0\,, \quad p\,(t, x_{fr}) = 0\,. \tag{3}$$

The problem formulated was solved numerically by the Godunov method [3, 8] and a grid characteristic method [9] developed for calculating quasi-one-dimensional flows in a hydraulic gun. The calculations were performed on grids of 256 and 512 cells. In the process of the calculations, fulfillment of the mass and energy balances was monitored. In the calculation on the fine grids, the mass disbalance did not exceed 0.05%, and that of the energy was no larger than 0.1%.

Let us briefly describe the difference scheme that was used in this work. We write the equations of quasi-one-dimensional is entropic motion of the fluid in the hydraulic gun in a Lagrangian representation in term of Riemann invariants [10]

$$\frac{d^+I}{dt} + a\,\frac{\partial^+I}{\partial x} = -\,\frac{ua}{F}\,\frac{dF}{dx}\,, \tag{4}$$

$$\frac{d^-I}{dt} - a\,\frac{\partial^-I}{\partial x} = \frac{ua}{F}\,\frac{dF}{dx}\,, \tag{5}$$

$$^{\pm}I = u \pm \frac{2}{n-1}\,a\,. \tag{6}$$

In Eqs. (4) and (5), the time derivative is total and is taken along the particle's trajectory.

We carry out the difference approximation of Eqs. (4) and (5) on a nonuniform moving Lagrangian grid whose nodes move together with the fluid. We mark the coordinates of the nodes and the parameters of the flow at them by integer indices that correspond to the numbers of the nodes: $i = 1, 2, ..., N$, by subscripts for the instant of time t and by superscripts after the time step Δt. The space derivatives will be approximated by left-hand or right-hand differences depending on the segment from which the values of the corresponding invariant are brought to the computational joint with the coordinate x^i: from the segment $[x_{i-1}, x_i]$ or the segment $[x_i, x_{i+1}]$. We average the coefficients of the derivatives and the free terms in the equations over the values of the parameters at the nodes of the required interval. We will mark them by half-integer indices: $i \pm 1/2$.

We will write the difference approximation of Eqs. (4) and (5) in the form

$$\frac{{}^{+}I^{i} - {}^{+}I_i}{\Delta t} + a_{i-1/2}\,\frac{{}^{+}I_i - {}^{+}I_{i-1}}{\Delta x_{i-1/2}} = -\left(\frac{ua}{\Delta x}\right)_{i-1/2} \ln \frac{F_i}{F_{i-1}}\,, \tag{7}$$

$$\frac{{}^{-}I^{i} - {}^{-}I_i}{\Delta t} + a_{i+1/2}\,\frac{{}^{-}I_{i+1} - {}^{-}I_i}{\Delta x_{i+1/2}} = -\left(\frac{ua}{\Delta x}\right)_{i+1/2} \ln \frac{F_{i+1}}{F_i}\,, \tag{8}$$

$$y_{i-1/2} = (y_{i-1} + y_i)/2\,, \quad \Delta x_{i-1/2} = x_i - x_{i-1}\,.$$

By replacing the invariants with their expressions and solving the system of equations for u^i and a^i, we find the parameters of the flow after a time step

$$u^i = u_i - \frac{\Delta t}{2}\,(D_{i-1/2} + D_{i+1/2})\,; \quad a^i = a_i - \frac{n-1}{4}\,\Delta t\,(D_{i-1/2} - D_{i+1/2})\,. \tag{9}$$

Here

$$D_{i-1/2} = \frac{a_{i-1/2}}{\Delta x_{i-1/2}}\left(u_i + \frac{2}{n-1}\,a_i - u_{i-1} - \frac{2}{n-1}\,a_{i-1}\right) + \left(\frac{ua}{\Delta x}\right)_{i-1/2} \ln \frac{F_i}{F_{i-1}}\,, \tag{10}$$

$$D_{i+1/2} = \frac{a_{i+1/2}}{\Delta x_{i+1/2}}\left(u_{i+1} - \frac{2}{n-1}\,a_{i+1} - u_i + \frac{2}{n-1}\,a_i\right) - \left(\frac{ua}{\Delta x}\right)_{i+1/2} \ln \frac{F_{i+1}}{F_i}\,. \tag{11}$$

The coordinates of the grid nodes after the time step are determined from the formula

$$x^i = x_i + \frac{u^i + u_i}{2}\,\Delta t\,. \tag{12}$$

The boundary points are calculated using the corresponding boundary conditions. For example, point N at the leading edge of the fluid, where the value of the velocity of sound is assigned, is calculated from the formulas

$$a^N = a_0\,, \quad u^N = u_N - D_{N-1/2}\Delta t\,, \quad x^N = x_N + \frac{u^N + u_N}{2}\,\Delta t\,. \tag{13}$$

The quantity $D_{N-1/2}$ is determined by expression (10).

The time step of the difference scheme is limited by the condition of Courant stability [10]

$$\Delta t \le \min\left(\frac{\Delta x}{a}\right)_{i+1/2}\,. \tag{14}$$

Test calculations showed sufficient efficiency and reliability of the considered difference scheme in solving a wide range of problems of nonstationary motion of a fluid. It can be noted that in terms of the organization of the algorithm and its speed of response the scheme is more efficient than the Godunov method, which has been widely used in numerical solution of problems of gas dynamics [11]. The scheme is not conservative, but the mass and energy balances, which were monitored in the calculations, were fulfilled

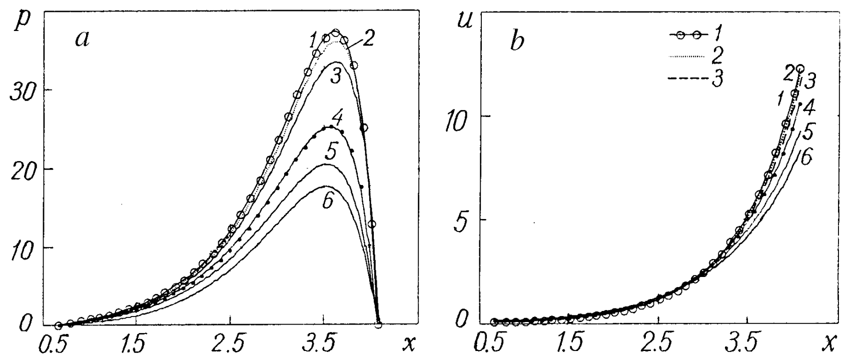

Fig. 2. Distribution of pressure (a) and velocity (b) along the coordinate x at the end of inflow for different values of the initial velocity: 1) incompressible fluid; 2–6) compressible fluid for an initial velocity of 25, 50, 150, 300, and 500 m/sec; dark points, calculation according to the Godunov method; light points, weakly compressible fluid. All the quantities are dimensionless.

with high accuracy at all stages of the process. One can mention among the scheme's drawbacks the necessity of singling out the positions of strong discontinuities and the possibility of strong deformation of the difference grid. In the presence of large velocity gradients, Lagrangian cells can be deformed significantly, which can lead to a loss of accuracy of the calculations. In the calculations, when the step of the cell exceeded the maximum, it was subdivided into two cells.

Results of calculations are given below for a hydraulic gun whose data were taken from [5]: the ratio of the length of the water charge to the length of the nozzle k_L = 1/3.07; the ratio of the areas of the inlet and outlet cross sections of the nozzle k_F = 100. All variables in the formulas and on the graphs are dimensionless. For simplicity, the same notation as for dimensional quantities is kept for them. When the occasion requires, the dimensions of the quantities are specified additionally. The selected scales are: the length of the water charge L, the area of the inlet cross section of the nozzle F_{in}, the initial velocity of the fluid U_0, the time L/U_0, and the velocity head $\rho U_0^2/2$. The profile of the nozzle changed by the exponential law

$$f(x) = \exp\left(-a_1 (x-1)\right),$$

where a_1 = 1.5. In [5], analytical expressions are obtained for the distribution of velocity and pressure along the coordinate in relation to the position of the leading edge. At the end of the inflow these expressions have the form

$$u(x) = u_{\text{re}} \exp\left(a_1 (x-1)\right), \tag{15}$$

$$p(x) = \begin{cases} \dfrac{(x - x_{\text{re}})(2\gamma - x_{\text{re}})\, x_{\text{re}}}{G^2}, & x_{\text{re}} \le x \le 1; \\[2ex] \dfrac{\gamma^2 (1 - 2x_{\text{re}})}{G^2} + \dfrac{\exp\left(\dfrac{x-1}{\gamma}\right)}{G}\left[\dfrac{x_{\text{re}}\, \gamma\, (2\gamma - x_{\text{re}})}{G} - (\gamma - x_{\text{re}}) \exp\left(\dfrac{x-1}{\gamma}\right)\right], & 1 \le x \le x_{\text{e}}. \end{cases} \tag{16}$$

Here

$$u_{re} = \left[\frac{(k_F - 1)^2}{a_1 k_F} + 1\right]^{-\frac{1}{2}}, \quad \gamma = \frac{1}{a_1}, \quad x_{re} = \frac{k_F - 1}{a_1 k_F}, \quad G = x_{re}^2 - x_{re} + \gamma .$$

For the maximum values of the velocity of outflow u_m and the pressure p_m approximate expressions are obtained for the case where the ratio of the areas of the inlet and outlet cross sections of the nozzle is large ($k_F >> 1$):

$$u_m = \sqrt{k_F k_L \ln k_F}, \quad p_m = u_m^2/4 . \tag{17}$$

Figure 2 shows the distribution of pressure and velocity along the coordinate x at the end of the fluid's inflow into the nozzle. Curves 1 correspond to an incompressible fluid and are constructed from formulas (5) and (4). Curves 2–6 are obtained in calculation of the flow of a compressible fluid according to the above-described difference scheme for different initial velocities of the fluid equalb to 25, 50, 150, 300, and 500 m/sec, respectively. The dark points on the graph are used to mark a numerical solution performed by the Godunov method for an initial velocity of 150 m/sec. As is seen, the coincidence of the results obtained in calculations by different numerical methods is complete. However, the consumption of computer time in the calculation by the grid-characteristic method is 4.5 times lower than by the Godunov method.

In Table 1, for different values of the fluid's initial velocity u_0, we give: the maximum value of the outflow velocity and the pressure for a compressible fluid u_m and p_m, the Mach number M corresponding to the maximum velocity u_m; the relative deviations of the maximum values of the velocity and the pressure δu and δp in comparison with the parameters of the flow of an incompressible fluid.

For an incompressible fluid the dimensionless values of the maximum pressure and velocity of the outflow do not depend on the initial velocity, and for this design of the hydraulic gun p_m = 37.5 and u_m = 12.28. For a compressible fluid the parameters of the flow depend noticeably on the initial velocity U_0, as follows from the table. The compressibility of the fluid exerts a stronger influence on the distribution and the maximum value of the pressure in the nozzle than on the distribution and the maximum value of the velocity. Taking into account the compressibility of the fluid leads to a noticeable reduction in the pressure and velocity of the outflow. For a compressible fluid for different values of the initial velocity the maximum pressure at the end of the inflow occurs in practically one and the same cross section with the coordinate $x_m \approx 3.6$. For an incompressible fluid the coordinate that corresponds to the maximum value of the pressure is determined by the approximate expression $x_m \approx 1 + \frac{1}{a_1} \ln \frac{k_F}{2}$ = 3.6 on condition that $k_F >> 1$. As is seen, the coincidence of the results for the character of the distribution of pressure for different models of the fluid is good.

According to expressions (17), the maximum pressure p_m of the incompressible fluid in the nozzle at the end of the inflow exceeds the velocity head u_m^2 fourfold (we remind the reader that the quantities are dimensionless). For a compressible fluid the maximum values of the pressure and the velocity are smaller than for an incompressible fluid and depend strongly on the initial velocity of the water charge. However the ratio of the maximum pressure to the velocity head is also close to four, which can readily be seen from the last column of the table. A maximum difference of 10% is observed for an initial velocity of 150 m/sec.

In [5, 6], the results of calculations for an incompressible fluid and a compressible one are compared. It is noted that the maximum difference in velocity is observed at the nozzle section and reaches 15%. The maximum values of the pressure practically do not differ. In the calculations performed, the maximum values of the pressure differ more than those of the velocity, which is in disagreement with the conclusions of [6]. For not very large values the pressure is proportional to the velocity squared. The error in the determination of pressure in the linear approximation will be twice as large as the error in the determination of velocity, i.e., it will be about 30%. This result agrees well with the data given in Table 1.

TABLE 1. Dependence of the Parameters of the Hydraulic Gun on the Initial Velocity of the Fluid

U_0, m/sec	u_m	δu	M	p_m	δp	$4p_m/u_m^2$
25	12.16	1	0.2	36	3.5	0.974
50	11.9	3.3	0.38	33.42	10.5	0.944
75	11.55	5.9	0.57	30.74	17.6	0.922
100	11.2	9.3	0.76	28.65	23	0.914
150	10.58	13.8	1.06	25.12	32.6	0.898
200	10.1	17.9	1.34	23.04	38.3	0.903
250	9.64	21.5	1.61	21.26	43.03	0.915
300	9.29	24.35	1.86	20.43	45.3	0.947
350	8.96	27.03	2.09	19.2	48.55	0.957

In [7], experimental investigations of a hydraulic gun are described and a comparison is made with the results of the calculations of [5, 6]. In the experiments, high-speed photography of the jet was done and its velocity was measured. The pressure inside the installation was not recorded. Based on the results of the experiment the conclusion is drawn that the fluid's compressibility can be disrefarded for jets with velocities up to 1500 m/sec. It can be seen from the table that for a velocity of outflow of 1500 m/sec (the M number ≈ 1) the difference in velocities for a compressible fluid and an incompressible one is relatively small – about 14%. The pressures differ much more – by 32%. With increase in the fluid velocity this difference increases even more. It can be noted that for the considered design of the hydraulic gun the theory of an incompressible fluid provides satisfactory coincidence in the outfow velocity and poor coincidence in the maximum pressure. The character of the flow, the distribution of the quantities in space, and the ratio of the characteristic parameters for different models of the fluid coincide satisfactorily.

Calculations were done for other values of the ratio of the areas k_F. The remaining geometric parameters and the initial velocity of the fluid were not changed. As should have been expected, with decrease in the area of the outlet cross section of the nozzle the maximum outflow velocity of the jet and the maximum pressure in the nozzle increased. For example, when the area ratio increases 1.5-fold ($k_F = 150$), the outflow velocity of the jet increases by 20% and the pressure increases by 60%. For this variant the coincidence of the results for an incompressible fluid and a compressible one is much poorer than for the variant considered above.

The Mach number M calculated from the maximum outflow velocity can be used as a criterion for permissibility to disregard the fluid's compressibility. It can be seen from the table that for the Mach number M = 0.38 the errors in determining the velocity and pressure amount to 3.3 and 10.5%, respectively, which is quite acceptable for engineering calculations.

It is known that the lower the compressibility of a fluid, the higher the velocity of sound in it. For an incompressible fluid the velocity of sound equals infinity. For ordinary water the velocity of sound at zero pressure, according to the Tate equation of state, is determined by the expression $a_0 = \sqrt{nB/\rho_0} \approx 1500$ m/sec. One can expect that as the velocity of sound a_0 increases, the results of the numerical calculations for a compressible fluid must converge to the solution for an incompressible fluid. In Fig. 2, such a solution for a weakly compressible fluid for an initial speed of sound of 20 a_0 (30 km/sec) is marked by light points. The initial velocity of the water is 150 m/sec. The required value of the velocity of sound is obtained by a formal 400-fold increase in the is enthropic exponent n in the equation of state of water. As is seen from the figure, the coincidence of the results for an incompressible fluid and a compressible one is excellent. Good coincidence occurs already for the velocity of sound equaling 10 a_0. The analysis made speaks, on the one hand, to the reliability of the calculation results and to the reliability of the algorithm. On the other hand, for a weakly compressible fluid with an assigned initial velocity the Mach number is small and the results of calculations of an incompressible fluid and a compressible fluid coincide well. This means that the Mach number can be taken as a criterion for assessing account for the compressibility of a fluid.

Thus, for an incompressible fluid the results do not depend on the initial velocity of the fluid. For a compressible fluid this dependence is substantial. The higher the initial velocity of the fluid, the greater the difference in the parameters of the flow for an incompressible fluid and a compressible one. Disregarding the fluid's compressibility yields too low values of the velocity and the pressure. The character of the change in the quantities is conveyed correctly. The Mach number calculated from the maximum velocity at the end of the inflow can be taken as a criterion for the permissibility of application of the incompressible-fluid model. When the Mach numbers are small, it is justified to disregard the compressibility of the fluid. When the Mach numbers are comparable to unity, it is necessary to take into account the fluid's compressibility.

NOTATION

a, velocity of sound; $a_0 = \sqrt{nB/\rho_0}$, velocity of sound at zero pressure; $a_1 = k_L \ln k_F$, parameter of the nozzle; B = 304.5 MPa, n = 7.15, $\rho_0 = 10^3$ kg/m^3, constants in the equation of state of water in the Tate form; $D_{i\pm1/2}$, auxiliary combinations; F, cross-sectional area of the nozzle; f, dimensionless cross-sectional area; G, auxiliary quantity; $^{\pm}I$, Riemann invariants; i, number of the grid node; k_L, ratio of the length of the water charge to the length of the nozzle; k_F, ratio of the areas of the inlet and outlet cross sections of the nozzle; L, length of the water charge; L_e, length of the nozzle; M, Mach number; N, number of nodes of the difference grid and number of the node on the front surface; p, pressure; t, time; Δt, time step of the difference scheme; u, velocity; U_0, initial velocity of the liquid; x, coordinate; y, function of the coordinate x; γ, coefficient; δp and δu, deviations of the pressure and the velocity; ρ, density. Subscripts: e and in, outlet (exit) and inlet of the nozzle; fr and r, front and rear surfaces of the water charge; i, number of the computational node of the grid; m, maximum value, re, rear surface of the water charge at the end of the inflow; 0, initial parameters.

REFERENCES

1. M. A. Lavrent'ev, É. A. Antonov, and B. V. Voitsekhovskii, *Problems of the Theory and Practice of Pulsed Water Jets* [in Russian], Novosibirsk (1961).
2. B. V. Voitsekhovskii, Yu. A. Dudin, Yu. A. Nikolaev, V. P. Nikolaev, and V. V. Nikitin, in: *Dinamika Sploshn. Sredy* (Novosibirsk), Issue 9 (1971), pp. 7–11.
3. G. A. Atanov, *Hydropulsed Installations for Destruction of Rocks* [in Russian], Kiev (1987).
4. I. L. Povkh and G. A. Atanov, in: *Gidromekhanika* (Kiev), Issue 15 (1969), pp. 69–76
5. J. L. Ryhming, *J. Appl. Math. Phys.* (ZAMP), **24**, 149–164 (1973).
6. L. A. Glenn, *Comput. Fluids*, **3**, 197–215 (1975).
7. B. Edney, in: *Proc. 3rd Int. Symp. Jet Cutting Technology* (Chicago, Illinois), Paper B2 (1976), pp. 11–26.
8. G. A. Atanov, V. I. Gubskii, and A. N. Semko, *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 6, 175–179 (1997).
9. G. A. Atanov, *Efficient Method for Numerical Solution of Unsteady Equations of Gas Dynamics*, Donetsk (1988), Dep. Ukr. NIINTI 10.08.88, No. 1879.
10. B. L. Rozhdestvenskii and N. N. Yanenko, *Systems of Quasilinear Equations and Their Applications to Gas Dynamics* [in Russian], Moscow (1976).
11. S. K. Godunov (ed.), *Numerical Solution of Multidimensional Problems of Gas Dynamics* [in Russian], Moscow (1976).